Mathematical Analysis - List 19

1. Sketch the region enclosed by the given curves and find its area.

a) $y^2 = 4x \;\; x + y = 3, \;\; y = 0, \; (y \geqslant 0)$; b) $x^2 + y^2 - 2y = 0, \;\; x^2 + y^2 - 4y = 0$;

c) $x^2 + y^2 = 2y, \;\; y = \sqrt{3}|x|$.

2. Find the volume of the solid bounded by the surfaces.

a) $x^2 + y^2 - 2y = 0, \;\; z = x^2 + y^2, \;\; z = 0$; b) $x^2 + y^2 + z^2 - 2z = 0$.

3. Find the area of the surface $x^2 + y^2 + z^2 = R^2$ bounded by $x^2 + y^2 - Rx \leqslant 0, \;\; z \geqslant 0$.

4. Find the area of the surface $z = x^2 + y^2$ bounded by $x^2 + y^2 \leqslant 1$.

5. Let f be a continuous function on V that is bounded by the given surfaces. Change $\iiint\limits_V f(x, y, z)\, dxdydz$ to an iterated integral.

a) $z = 2\sqrt{x^2 + y^2}, \;\; z = 6$; b) $x^2 + y^2 + z^2 = 25, \;\; z = 4 \; (z \geqslant 4)$;

c) $z = x^2 + y^2, \;\; z = \sqrt{20 - x^2 - y^2}$.

6. Find new limits of integration.

a) $\int\limits_0^1 dx \int\limits_0^{2-2x} dy \int\limits_0^{3-3x-1.5y} f(x, y, z)\, dz = \int dz \int dx \int f(x, y, z)\, dy$;

b) $\int\limits_{-2}^2 dx \int\limits_{-\sqrt{4-x^2}}^0 dy \int\limits_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} f(x, y, z)\, dz = \int dz \int dy \int f(x, y, z)\, dx$;

c) $\int\limits_0^3 dz \int\limits_{-\sqrt{z}}^{\sqrt{z}} dx \int\limits_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} f(x, y, z)\, dy = \int dy \int dz \int f(x, y, z)\, dx$;

d) $\int\limits_0^1 dx \int\limits_0^{\sqrt{1-x^2}} dy \int\limits_{x^2+y^2}^1 f(x, y, z)\, dz = \int dz \int dx \int f(x, y, z)\, dy$.

7. Evaluate the triple integrals in cylindrical coordinates.

a) $\iiint\limits_V (x^2 + y^2 + z^2)\, dxdydz, \qquad V: \;\; x^2 + y^2 \leqslant 4, \;\; 0 \leqslant z \leqslant 1$;

b) $\iiint\limits_V xyz\, dxdydz, \qquad V: \;\; \sqrt{x^2 + y^2} \leqslant z \leqslant \sqrt{1 - x^2 - y^2}$;

c) $\iiint\limits_V (x^2 + y^2)\, dxdydz, \qquad V: \;\; x^2 + y^2 + z^2 \leqslant R^2, \;\; x^2 + y^2 + z^2 \leqslant 2Rz$.

8. Evaluate the triple integrals in spherical coordinates.

a) $\iiint\limits_V \dfrac{dxdydz}{\sqrt{x^2+y^2+z^2}}\,, \qquad V:\ 4 \leqslant x^2+y^2+z^2 \leqslant 9;$

b) $\iiint\limits_V (x^2+y^2)\,dxdydz, \qquad V:\ \sqrt{x^2+y^2} \leqslant z \leqslant \sqrt{1-x^2-y^2};$

c) $\iiint\limits_V z^2\,dxdydz, \qquad V:\ x^2+y^2+(z-R)^2 \leqslant R^2,\ \ R>0;$

d) $\iiint\limits_V x^2\,dxdydz, \qquad V:\ x^2+y^2+z^2 \leqslant 4x.$